

AFHRL-TR-79-70

# AIR FORCE

MAN RESOURCES

LEVCI 1



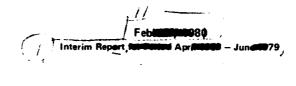
ITEM CHARACTERISTIC CURVE PARAMETERS:
EFFECTS OF SAMPLE SIZE ON LINEAR EQUATING.

Mal

Malcolm James Ree Harald E. Jensen

PERSONNEL RESEARCH DIVISION
Brooks Air Force Base, Texas 78235

12 1-



Approved for public release; distribution unlimited.

DTIC ELECTE MAR 27 1980

**LABORATORY** 

AIR FORCE SYSTEMS COMMAND

**BROOKS AIR FORCE BASE, TEXAS 78235** 

30 3 21

078

#### NOTICE

When U.S. Government drawings, specifications, or other data are used for any purpose other than a definitely related Government procurement operation, the Government thereby incurs no responsibility nor any obligation whatsoever, and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise, as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

This interim report was submitted by Personnel Research Division, under project 7719, with HQ Air Force Human Resources Laboratory (AFSC), Brooks Air Force Base, Texas 78235. Dr. Malcolm James Ree was the Principal Investigator for the Laboratory.

This report has been reviewed by the Office of Public Affairs (PA) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

NANCY GUINN, Technical Director Personnel Research Division

RONALD W. TERRY, Colonel, USAF Commander

Unclassified
SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered)

	READ INSTRUCTIONS BEFORE COMPLETING FORM	
	GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
AFHRL-TR-79-70		
4. TITLE (and Subtitle)		5. TYPE OF REPORT & PERIOD COVERED
ITEM CHARACTERISTIC CURVE PARAMETERS:		Interim
EFFECTS OF SAMPLE SIZE ON LINEAR EQUATING		April 1979 – June 1979
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(a)		8. CONTRACT OR GRANT NUMBER(s)
Malcolm James Ree Harald E. Jensen		
riaraid E. Jensen		
PERFORMING ORGANIZATION NAME AND ADDRESS		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
Personnel Research Division		AREA & WORK UNIT NUMBERS
Air Force Human Resources Laboratory		62703F
Brooks Air Force Base, Texas 78235		77191522
1. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE
HQ Air Force Human Resources Laboratory (AFSC)		February 1980
Brooks Air Force Base, Texas 78235		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS(If different i	from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE
6. DISTRIBUTION STATEMENT (of this Report)		
<ol> <li>DISTRIBUTION STATEMENT (of the abatract entered in</li> </ol>	Block 20, if different from	n Report)
8. SUPPLEMENTARY NOTES		
6. SUPPLEMENTARY NOTES		
	identify by block number)	
9. KEY WORDS (Continue on reverse side if necessary and item analysis	identify by block number) computer adaptive t	esting
9. KEY WORDS (Continue on reverse side if necessary and item analysis latent-trait theory	computer adaptive t equating	-
9. KEY WORDS (Continue on reverse side if necessary and item analysis latent-trait theory sample size	computer adaptive t	-
9. KEY WORDS (Continue on reverse side if necessary and item analysis latent-trait theory sample size simulations	computer adaptive t equating	-
9. KEY WORDS (Continue on reverse side II necessary and item analysis latent-trait theory sample size simple size additional and sample size additions adaptive testing	computer adaptive t equating item characteristic c	-
9. KEY WORDS (Continue on reverse side if necessary and item analysis latent-trait theory sample size simulations adaptive testing	computer adaptive t equating item characteristic c	urve
9. KEY WORDS (Continue on reverse side if necessary and item analysis latent-trait theory sample size simulations adaptive testing  O DESTRACT (Continue on reverse side if necessary and it in order to establish large item pools, it is not because it is impractical to administer several hundre	computer adaptive t equating item characteristic c dentity by block number) ecessary to try out hu d items to several thous	urve  ndreds of items for possible inclusion. sand people, the invariance properties of
9. KEY WORDS (Continue on reverse side if necessary and item analysis latent-trait theory sample size simulations adaptive testing  O DESTRACT (Continue on reverse side if necessary and it in order to establish large item pools, it is not because it is impractical to administer several hundre Item Characteristic Curve (ICC) parameters are used	computer adaptive t equating item characteristic c dentity by block number? eccssary to try out hu d items to several thous d to advantage to equ	ndreds of items for possible inclusion. sand people, the invariance properties of ate test items. A simulation study was
9. KEY WORDS (Continue on reverse side if necessary and item analysis latent-trait theory sample size simulations adaptive testing  O DESTRACT (Continue on reverse side if necessary and it in order to establish large item pools, it is not because it is impractical to administer several hundre litem Characteristic Curve (ICC) parameters are used conducted in order to investigate the effects of errors.	computer adaptive t equating item characteristic c dentity by block number; ecessary to try out hu d items to several thou d to advantage to equi ors in estimated ICC p	ndreds of items for possible inclusion. sand people, the invariance properties of ate test items. A simulation study was arameters on equated parameters. Four
9. KEY WORDS (Continue on reverse side if necessary and item analysis latent-trait theory sample size simulations adaptive testing  O DESTRACT (Continue on reverse side if necessary and it in order to establish large item pools, it is not because it is impractical to administer several hundre Item Characteristic Curve (ICC) parameters are used	computer adaptive t equating item characteristic condensity by block numbers ecessary to try out hurd items to several thoused to advantage to equipors in estimated ICC pro 2,000 subjects, were	ndreds of items for possible inclusion. sand people, the invariance properties of ate test items. A simulation study was arameters on equated parameters. Four administered a different test containing

and equating same size.

DD 1 FORM 1473

Unclassified

SECU	RITY CLASSIFICATION OF THIS	PAGE(When Date Enter	· ·d)	· · · · · · · · · · · · · · · · · · ·	<del></del>
					i
1					
]					
ł					
l					
}					
1					
İ					
1					
}					
					j
1					
}					
					ĺ
1					
}					
1					
1					
1					

## **PREFACE**

This research was conducted under Project 7719, Air Force Development of Selection, Assignment, Performance Evaluation, Retention and Utilization Devices; Task 771915, Perceptual and Computer Managed Measurement.

The authors wish to express their appreciation to Dr. Janos Koplyay, Air Force Human Resources Laboratory and to Mr. Thomas Warm, U.S. Coast Guard Institute, for their critical review of the effort.

Acces				
	NTIS GRA&I			
DDC T		H		
	ounced	$\Box$		
Justi	fication_	<u> </u>		
Ву				
Distr:	ibut.ion/			
<u> </u>	<b>♦</b> * *** •	odes		
	Avail and	or		
Dist	Dist   special			
	2			
H		į.		

# TABLE OF CONTENTS

l.	Introduction	Page 5
	ICC Parameters	5 7
11.	Method	7
	Generation of Item Responses	8
111.	Results	9
IV.	Discussion	13
	Estimating and Equating a	13 13 14
Refe	rences	14
	LIST OF ILLUSTRATIONS	
Figure		Page
l l	Item characteristic curves	6
2	Error in Estimation of ICC Parameter	11
	LIST OF TABLES	
Table ]	Mean, Standard Deviation, Minimum and Maximum of $\theta$ for Groups S1 and S2	Page 8
2	Means and Standard Deviations of the Generated Item Parameters for Test 1 (T1) and Test 2 (T2)	8
3	ICC Parameters of the 20 Anchor Items Common to Both Tests	9
4	Intercorrelations between Known and Estimated ICC Parameters for Both Groups with Varying Sample Sizes	10
5	Summed Absolute Deviations (\(\mathbb{E}\)[Error\) and Averaged Absolute  Deviations (\(\begin{align*} \text{Error}\)\) for the Three ICC Parameters for the Two Tests	10
6	Summed Absolute Deviations (\(\Sigma\)[Error\) and Average Absolute Deviations (\(\sigma\)[Error\) for the a and be Parameters for Various Equating and Calibrating Sample Sizes	12
7	Intercorrelations, Means, and Standard Deviation of the Estimated  a Parameters for Test 2	12
8	Intercorrelations, Means, and Standard Deviation of the Estimated	12

# ITEM CHARACTERISTIC CURVE PARAMETERS: EFFECTS OF SAMPLE SIZE ON LINEAR EQUATING

#### I. INTRODUCTION

The application of the technology of computer driven adaptive testing requires the development of large banks of test items. Each bank may contain 250 to 400 items, and all must measure the same ability on the same metric or scale. It is unreasonable and impracticable to assemble a single group of 2,000 subjects for 250 to 400 minutes to try all the items; therefore, a method for linking together subsets of items administered to varying groups must be investigated. Item Characteristic Curve (ICC) theory offers a unique method of linking subsets of test items due to the invariance property of the ICC parameters. This invariance property rests on the two major theoretical assumptions of latent-trait theory: (a) unidimensionality and (b) local independence. Unidimensionality means that only a single ability is being measured and is assumed to be the property of an item pool, even when assembled into subsets. Local independence means that the subjects' responses to an item are independent of the responses to another item. More simply put, this means that the item response is a function of ability and no other factor. In effect, this is a restatement of the unidimensionality assumption. If an item pool is unidimensional, then any shift in score metric that is due to a linear transformation may be corrected or adjusted by application of the proper complementary linear transformation. This is what is meant by the idea that latent-trait parameters are invariant to a linear transformation, and it is this theoretical property that allows item pools to be linked and transformed to a common metric. In previous research efforts, item pools have been linked via the method of linear equating (see Lord, 1977; Ree, 1977; Sympson & Ree, in press) with apparent success. To date, there has been little research on the efficacy of these linking procedures and the effects of errors in ICC parameter estimation on their (linearly) transformed values.

# **ICC Parameters**

Michael Carlo Transport

The three parameter logistic model of Birnbaum (Lord & Novick, 1968) is the most frequently used for relating item responses to subjects' ability. The three parameters, a, b, and c, are item discrimination, item difficulty (or location), and probability of chance success (or lower asymptote), respectively.

The curve described by these parameters takes the shape of an ogive (cumulative frequency) or an "s" with the upper asymptote approaching a probability of 1.0 and usually a lower asymptote of a probability greater than 0.0. The ogive describes the probability of obtaining a correct answer to an item as a monotonic increasing function of ability.

The item discrimination parameter, a, is a function of the slope of the ICC and generally ranges from .5 to about 2.5. The value of a equal to about 1.0 is typical of many test items, while a values below .5 are insufficiently discriminating for most testing purposes, and a values above 2.0 are infrequently found.

The item difficulty parameter, b, describes the point of inflection of the ICC and is usually scaled between -2.5 and +2.5, although the metric is arbitrary.

The item guessing parameter, c, is the lower asymptote of the ICC and is generally conceived as the probability of selecting the correct item-option by chance alone. Most test items have c parameters greater than 0.0 and less than or equal to .30.

Figure 1 shows three ICCs. The horizontal axis is scaled in units of ability  $\theta$  and the vertical axis is the probability of answering the item correctly. The solid curved line shows an ICC

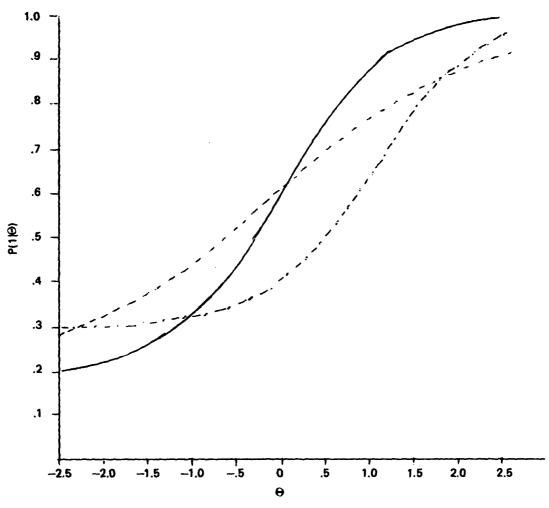


Figure 1. Item characteristic curves.

for an item of average difficulty with acceptable discrimination and the lower asymptote appropriate for a five-item multiple-choice item. The dashed line shows an item of identical difficulty, c value of .28, but with a lower a value. Note how the slope of the curve is less steep. The third curve, dot-dash line, shows an item with a c value of .30, an a parameter of 1.0, and the b parameter equal to 1.0. As the b parameter changes, the location of the inflection point of the curve is displaced along the horizontal axis.

Equation 1 presents the mathematical function describing the curve.

$$p(\theta)_{j} = c_{i} + (1 - c_{i}) \left(1 + e^{\left(-1.7a_{i}(\theta - b_{i})\right)}\right)^{-1}$$
 (1)

Previous research (Ree, 1978) indicates that the ICC parameters may be estimated with some reasonable degree of accuracy, providing a sufficient sample of examinees with an appropriate distribution of ability,  $\theta$  is available.

#### Linking Paradigms

Two fundamental linking procedures may be defined and are known as the Anchor Items Method (AIM) and the Anchor Subjects Method (ASM). In AIM, every subset of items is administered to a different sample of subjects, but embedded into the group of items to be analyzed is a common (or anchor) set of items. During analysis, the anchor items are identified, and the following linear transformation is applied to the resultant ICC parameters:

$$b_{t} = \left(\frac{sb_{t}}{sb_{2}}\right) \quad b_{2} + \left[\overline{b}_{t} - \left(\frac{sb_{t}}{sb_{2}}\right) - \overline{b}_{2}\right]$$
 (2)

Where  $b_t$  is the item location parameter transformed to the desired scale and  $sb_t$  and  $sb_2$  are standard deviations of the desired scale and observed scale respectively. A similar procedure for the a parameter is defined by

$$a_t = a_2 \cdot \frac{sb_2}{sb_t} \tag{3}$$

Where  $a_t$  is the item discrimination parameter transformed to the desired scale,  $a_2$  is the observed a parameter, and  $sb_t$  and  $sb_2$  are as in equation (2). Because the c parameter is measured on the probability axis, it does not change and no transformation need be applied.

The ASM requires that the same group of subjects be available to take each subset of items. It is extremely unlikely that the same 2,000 subjects could be assembled to take items over a long period of time as would be required to place tests on the same metric from year to year. For this reason, the ASM method seems less likely to find long-term practical application. Because of its potential for use, the AIM procedure is the subject of the present study.

#### II. METHOD

In order to have a known standard for reference, a simulation study was run using two groups of subjects, a single set of 20 anchor items and two differing groups of 60 experimental, or non-anchor, items. These two groups of items were assembled into two tests designated Tl and T2. Both groups of simulated subjects were specified to have about the same normal distribution of  $\theta$ . Table 1 shows the mean, standard deviation, minimum and maximum of  $\theta$  for the groups Sl and S2. These two groups represent what might be expected if subjects for experimental testing were picked from some larger pool, such as candidates for military enlistment for example. Response vectors for these subjects were generated on the two tests.

Table 1. Mean, Standard Deviation, Minimum and Maximum of 0 for Groups S1 and S2

	Gro	u ps
Parameter	<b>S</b> 1	\$2
$\bar{x}_{\theta}$	0.0145	0.0250
$\sigma_{m{ heta}}$	0.9976	1.0045
Minimum	-2.6000	-2.6000
Maximum	2.6000	2.6000

#### Generation of Item Responses

In order to generate a vector of item responses for each "subject" the  $\theta$  values were used in equation (1) to compute the likelihood of "passing" each item.

Because Equation 1 yields a number  $P(\theta)_j$  such that  $0.0 < P(\theta)_j < 1.0$ , a number  $X_j$  is drawn from a uniform (rectangular) distribution ranging from 0.0 to 1.0 and compared to  $P(\theta)_j$ . If  $X_j$  is larger than  $P(\theta)_j$ , then an incorrect response is specified for the item; otherwise, a correct response is specified for the item. Thus, a "subject" with  $P(\theta)_j = .90$  gets the item correct 9 in 10 times, and a vector of item responses is developed for each "subject" in each data set. These response vectors are then used to investigate the AIM linking procedures.

Table 2 shows the distribution of ICC parameters for the 80 items for Test 1 (T1) and Test 2 (T2), while Table 3 shows the ICC parameters for the 20 anchor items which are common to both tests.

Subjects from Group 1 were administered only the items in Test 1, and subjects from Group 2 only the items in Test 2. In order to study the effects of sample size, the ICC parameters were estimated on four samples drawn with replacement as follows: 250; 500; 1,000; and 2,000. The ICC parameters were estimated on these four sample sizes for both groups. Anchor ICC parameter values from the four samples administered Test 1 serve as the input values for the anchor item parameters to the second test. This permitted the four sizes of calibration sample (250; 500; 1,000; 2,000) to be varied and tried out with the four samples used to estimate the anchor item ICC parameters.

Table 2. Means and Standard Deviations of the Generated Item Parameters for Test 1 (T1) and Test 2 (T2)

	Test	est
Parameter	Tŧ	T2
<del>a</del>	1.0564	1.0452
$\sigma_{\mathbf{a}}$	0.2793	0.2394
Б	0.0847	0.0559
$\frac{\sigma_{\mathbf{b}}}{\bar{c}}$	0.8442	0.8577
ē	0.1878	0.2017
$\sigma_{c}$	0.0542	0.0474

Table 3. ICC Parameters of the 20 Anchor Items Common to Both Tests

		ICC Parameter	
Number		b	c
1	.8000	-1.5000	.1000
2	.8000	-1.3500	.1000
3	1.0000	-1.2000	.1500
4	1.0000	-1.0500	.1500
5	1.1000	9000	.2000
6	1.2000	<b>7500</b>	.2000
7	1.2000	6000	.2200
8	1.2000	<b>4500</b>	.2000
9	1.3000	3000	.2000
10	1.4000	1500	.2000
11	1.4000	.1500	.2200
12	1.3000	.3000	.2500
13	1.2000	.4500	.2000
14	1.2000	.6000	.2200
15	1.1000	.7500	.2200
16	1.0000	.9000	.2000
17	1.0000	1.0500	.2500
18	.8000	1.2500	.2500
19	.8000	1.3500	.2500
20	.8000	1.5000	.2500
Mean	1.0600	.0000	.2015
SD	.2113	.9549	.0453

## III. RESULTS

Table 4 shows the intercorrelations between the known item parameters and the estimated parameters. As past research indicates (Urry, 1976), the correlations all increase with increasing sample size. The correlations in Test 1 for b and estimates of b start high at .952 and increase to an exceptionally high .992. Correlations for a and estimates of a begin moderately at .666 and climb to .869, but the correlations of c and estimated c increase from only .031 to .115. In Test 2, much the same pattern is observed except that the correlation of c and estimated c increases from .164 to .315 as sample size increases.

Because correlations are insensitive to constant differences as might be found if ICC parameters are overestimated or underestimated by a constant amount, summed absolute deviates of the estimated parameters from the known parameters were computed for each parameter in each sample size. Table 5 presents the summed absolute deviations (or summed errors) for both tests with the four sample sizes. Figure 2 displays this graphically. There is a large drop in summed error when the a parameter is estimated on progressively larger samples of subjects up to and including the difference between 1,000 and 500 subjects. Between 1,000 and 2,000 subjects, the difference in summed error is smaller. The relationship between error and sample size for the b parameter is more nearly constant. That is, the line on the figure for estimates of b is generally straight which means error tends to be reduced in direct proportion to the number of subjects. The almost flat line for the c parameter indicates that virtually no reduction of error is occurring

Table 4. Intercorrelations between Known and Estimated ICC Parameters for Both Groups with Varying Sample Sizes

Parameter	N	Test 1	Test 2
a	250	.666	.512
	500	.671	.725
	1,000	.831	.813
	2,000	.869	.886
b	250	.952	.929
	500	.964	.962
	1,000	.980	.979
	2,000	.992	.987
С	250	.031	.164
	500	.035	.109
	1,000	012	.331
	2,000	.115	.315

Table 5. Summed Absolute Deviations (Σ|Error|) and Average Absolute
Deviations (|Error|) for the Three ICC Parameters
for the Two Tests

		Test 1		Test 2	
Parameter	N	Σ Error	Errori	Σ Error	Error
a	250	30.6450	.3831	30.5290	.3816
a	500	22.8090	.2851	20.6910	.2586
a	1,000	15.7490	.1969	16.8910	.2111
a	2,000	15.5980	.1950	15.1390	.1892
b	250	23.5050	.2938	20.8470	.2606
b	500	19.8600	.2483	16.6070	.2076
b	1,000	17.6890	.2211	13.8050	.1726
b	2,000	12.7350	.1592	11.5130	.1439
c	250	7.7360	.0967	7.2350	.0904
c	500	7.3600	.0920	7.5120	.0939
¢	1,000	6.9080	.0864	7.3180	.0915
c	2,000	6.4400	.0805	6.8640	.0858

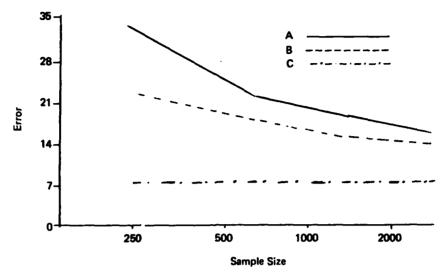


Figure 2. Error in Estimation of ICC Parameter.

with increasing sample size for that parameter. The average absolute deviation for the c parameter is almost one-third of the entire range of the parameter as the c parameter is generally estimated between .00 and .30. However, past research (Ree, 1979) indicates that, even for low ability subjects, the effects of errors in the estimation of the c parameter are small.

Summed deviations of known ICC parameters from the equated value of the ICC parameters were computed for the a and b parameters for the 16 combinations of calibration sample size and equating sample size. Table 6 shows the summed deviations and the per item deviation for both parameters for the 16 combinations. The equated a parameter shows large summed deviations whenever the sample has been limited to 250 subjects whether in the calibration or equating sample. The lowest error rates for the a parameter occur when the anchor item values have been estimated on 2,000 subjects. The effects of the size of the calibration sample are not so clear-cut. When 2,000 subjects are used to estimate the anchor item ICC parameters, the magnitude of the error is approximately the same for all calibration sample sizes except 250. With increasing calibration sample size, the error rate increases by some small amount as indicated by the average (per item) error. This is an unexpected result and an explanation may be found in the relationship between the sets of estimated a parameters. If the estimated a parameters were all estimates of the same value and if the test scale were unidimensional, a basic assumption of the theory, then the estimated a parameters should be linear transformations of one another and should be correlated 1.0, as correlations are invariant to a linear transformation. This was not found to be the case, and Table 7 shows the intercorrelation of the estimated a parameters. Only the correlation between the estimate of a calculated on 1.000 subjects and the estimate  $\beta/a$ calculated on 2,000 subjects approaches this relationship. This lack of linearity may be due to the assumption of normality and to the rescaling used in the calibration procedure, and these was interact in such a way as to produce the anomalous results. Table 8 shows the array or erat or estimated b parameters. All exceed .900, and the summed deviations also show a steady is case as sample size increases for the h parameter indicating a syrtially linear transform

Table 6. Summed Absolute Deviations (Σ|Error|) and Average Absolute Deviations (|Error|) for the a and b Parameters for Various Equating and Calibrating Sample Sizes

			Parai	neter		
Number of	Subjects		a			
Calibration	Equating	ΣlErrorl	iErrori	LiErrori	Error	
250	2000	34.2263	.4278	23.3679	.2921	
500	2000	15.1282	.1891	21.9342	.2742	
1000	2000	15.9871	.1998	16.3660	.2046	
2000	2000	16.5958	.2074	13.4579	.1682	
250	1000	38.3625	.4795	25.6440	.3205	
500	1000	17.6788	.2210	24.3413	.3043	
1000	1000	19.5867	.2448	19.1156	.2389	
2000	1000	21.0321	.2629	16.8828	2110	
250	500	48.6112	.6076	25.4374	.3180	
500	500	24.5582	.3070	22.8994	.2862	
1000	500	28.8291	.3604	18.1871	.2273	
2000	500	31.2094	.3901	15.8328	.1979	
250	250	44.3122	.5539	26.2011	.3275	
500	250	21.5767	.2697	24.4160	.305	
1000	250	24.4389	.3117	19,4843	.2436	
2000	250	27.0242	.3378	17.3255	.2166	

Table 7. Intercorrelations, Means, and Standard Deviation of the Estimated a Parameters\* for Test 2

===				
	1		3	4
1	1.000			
2	.757	1.000		
3	.690	.860	1.000	
4	.595	.803	.926	1.000
Mean	1.3525	1.2539	1.2348	1.2268
SD	.4843	.3347	.3254	.3061

 $<sup>^{\</sup>rm a}{\rm Variables}$  are for the four sample sizes: 250; 500; 1,000; 2,000.

Table 8. Intercorrelations, Means, and Standard Deviation of the Estimated b Parameters<sup>a</sup> for Test 2

	1	2	3	
1	1.00			
2	.952	1.00		
3	.940	.978	1.00	
4	.935	.969	.986	1.00
Mean	.0563	.0591	.0735	.0559
SD	.8558	.8384	.8700	.8727

<sup>&</sup>lt;sup>a</sup>Variables are for the four sample sizes: 250; 500: 1,000; 2.000.

estimated b parameters from sample to sample. However, with 500 subjects in the equating sample, a similar anomaly is observed which may also be due to normal assumptions and to rescaling.

#### IV. DISCUSSION

The results of the study present new evidence of the critical interrelationship between item calibration and equating sample sizes and the values of ICC parameters.

# Estimating and Equating a

For the 16 combinations of calibration sample sizes and equating sample sizes identified in Table 6, the least deviation of estimated a from its known value occurred with an equating sample size of 2,000 and a calibration sample size of 500. As mentioned in the previous section, although the least error between the estimated and known a values was expected with a match of 2,000 equating and 2,000 calibrating sample sizes, the error actually increased very slightly with increasing calibration sample sizes beyond 500. This discrepancy apparently results from a non-linear transformation with sample sizes of 250 and 500 but tends toward linearity with sample sizes of 1,000 and 2,000.

During equating procedures, a sample size > 500 should be developed to ensure an acceptable degree of confidence that the estimation of a does not significantly depart from its "true" value. In the same light, estimation of a suffers considerably using equating sample sizes of less than 500 such that equating samples of 1,000 or 2,000 are highly desirable to minimize error in estimating a.

#### Estimating and Equating b

Table 6 also shows the linear relationship between error and sample size for the b parameter. The b parameter is best estimated with calibration and equating samples of 2,000 each, although a calibration sample size of 1,000 with an equating sample size of 500 can be tolerated without an appreciable increase in error. With all combinations of calibration and equating sample sizes, b is estimated quite well.

#### Estimating and Equating c

The flat line drawn in Figure 2, representing the data from Table 5, shows the estimation of the c parameter to be nearly insensitive to increases in sample size. As sample size increases from 250 to 2,000 subjects, the error decreases but only very slightly. With the c defined as the lower asymptote of the ICC and representing the probability of extremely low ability examinees correctly answering an item, the inability to estimate c with precision could be disturbing. However, it has been pointed out (Lord, 1975) that if a  $(\theta \cdot b) < -2$ , then the probability of a correct response is c. Therefore, if there are a large number of subjects with ability  $\theta$  so that  $\theta < -(2/a - b)$ , c can be accurately estimated. If this requirement is not met, c will be poorly estimated.

A stable and accurate estimate of the a and b parameters requires large numbers of subjects over a broad range of ability. The estimation of c requires large numbers of subjects at very low ability levels. This holds for both equating and calibrating samples; therefore, it is necessary to administer test items, whether to be calibrated or equated, to the largest samples available.

#### REFERENCES

- Lord, F.M. Evaluation with artificial data of a procedure for estimating ability and item characteristic curve parameters. ETS-RB-75-33. Princeton, NJ: Educational Service, 1975.
- Lord, F.M. Practical applications of item characteristic curve theory. *Journal of Educational Measurement*, 1977, 14, 117-138.
- Lord, F.M., & Novick, M.R. Statistical theories of mental test scores. Reading, MA: Addison-Wesley, 1968.
- Ree, M.J. Adaptive testing at an AFEES. Paper delivered at the Annual Meeting of the Military Testing Association, San Antonio, Texas, 1977.
- Ree, M.J. Estimating item characteristic curves. AFHRL-TR-78-68, AD-A064 739. Brooks AFB, TX: Personnel Research Division. Air Force Human Resources Laboratory, November 1978.
- Ree, M.J. The effects of errors in the estimation of item characteristic curve parameters. Paper presented at the Annual Meeting of the Military Testing Association, San Diego, California, 1979.
- Sympson, B., & Ree, M.J. A validity comparison of adaptive testing in a military technical training environment. Brooks AFB, TX: Air Force Human Resources Laboratory, Manpower and Pesonnel Research Division, in press.
- Urry, V. A five-year quest: Is computerized adaptive testing feasible? Proceedings of the First Conference on Computerized Adaptive Testing, Washington D.C.: Government Printing Office, March 1976.